# Search for $B_c(ns)$ via the $B_c(ns) \to B_c(ms)\pi^+\pi^-$ transition at LHCb and $Z_0$ factory

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# Abstract:

It is interesting to study the characteristics of the whole fam-

ily of  $B_c$  which contains two different heavy flavors. LHC and the proposed  $Z^0$  factory provide an opportunity because a large database on the  $B_c$  family will be achieved.  $B_c$  and its excited states can be identified via their decay modes. As suggested by experimentalists,  $B_c^*(ns) \to B_c + \gamma$  is not easy to be clearly measured, instead, the trajectories of  $\pi^+$  and  $\pi^-$  occurring in the decay of  $B_c(ns) \to B_c(ms) + \pi^+\pi^-$  (n > m) can be unambiguously identified, thus the measurement seems easier and more reliable, therefore this mode is more favorable at early running stage of LHCb and the proposed  $Z^0$  factory. In this work, we calculate the rate of  $B_c(ns) \to B_c(ms) + \pi^+\pi^-$  in terms of the QCD multipole-expansion and the numerical results indicate that the experimental measurements with the luminosity of LHC and  $Z^0$  factory are feasible.

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#### I. INTRODUCTION

The heavy quarkonia such as charmonia and bottomia have been experimentally and theoretically explored for several decades already because large database on them is available. In comparison, the physics on  $B_c$  has not been thoroughly studied yet. The reason is obvious that  $B_c$  contains two different heavy flavors, so unlike quarkona, it is produced via more suppressed processes at  $e^+e^-$  colliders. The earlier work [1] indicates that at the luminosity of regular  $e^+e^-$  colliders (the LEP I and II), one cannot expect to observe  $B_c$  production, i.e. its production rate is too small to be measured. Therefore, people should turn to hadron colliders. As predicted [1],  $B_c$  was observed at TEVATRON[2] a while ago. At the energy and luminosity of LHC, one may anticipate  $B_c$  events thousand times more than at TEVATRON. Now a project of constructing a  $Z^0$  factory is proposed which will provide sufficiently high luminosity at the  $Z^0$  pole. Even though the production process  $e^+e^- \to B_c + \overline{B_c}$  where a pair of heavy quarks ( $c\bar{c}$  or  $b\bar{b}$ ) emerges from a hard gluon emission, is suppressed, the high luminosity and the pole effect would greatly enhance the production rate, i.e. the enhancement compensates the suppression and enables the production of  $B_c$  measurable.

Since  $B_c$  is made of two different heavy flavors, its decay characteristics would somehow distinct from the heavy quarkonia which contains a pair of heavy quarks of the same flavor. Namely, the two constituents in quarkonia annihilate into gluons which afterwards hadronize. Although this mode is OZI suppressed, it is a strong interaction-induced process and has a larger decay width. Nevertheless the ground state of  $B_c$  family can only decay via weak interaction and its lifetime has been carefully studied[3]. It is interesting to investigate such mesons and we are not only interested in the ground state, but also its excited states. The lowest excited states would be the vector  $B_c^*$  (1s) and pseudoscalar  $B_c(2s)$  (0<sup>-</sup>) and the latter one is the first radially excited state of the family. A simple analysis estimates the splitting between  $B_c(1s)$  and  $B_c^*(1s)$  is about 50 to 80 MeV, so that  $B_C^* \to B_c + \pi^0$  is forbidden by the energy-conservation and the hadronic decays of  $B_c^*$  can only occur via weak processes which are obviously suppressed. Only possible transition is the radiative decay  $B_c^* \to B_c + \gamma$ , but at LHC, detection of a single photon from a messy background is very difficult. Instead, in the decay  $B_c(ns) \to B_c(ms) + \pi^+\pi^-$  the daughter charged pions are

very easy to be identified. Therefore, our experimental colleagues strongly suggest us to investigate the channel  $B_c(ns) \to B_c(ms) + \pi^+\pi^-$ . Definitely, we can gain more information about the  $B_c$  family.

Since  $B_c$  is composed of two heavy quarks, the relativistic effects may not be too serious, thus the potential model can be a good choice for determining the spectra of  $B_c$  and its excited states. In analog to dealing with heavy charmonia and bottomia, we employ the Cornell potential to calculate the masses of  $B_c(ns)$  and  $B_c^*(ns)$  where the mass of the ground state  $B_c(1s)$  is taken as input for fixing concerned model parameters.

Following literature [4–8] we evaluate the decay rate of  $B_c(ns) \to B_c(ms) + \pi\pi$  in terms of the QCD multipole-expansion. The picture is depicted as that the initial  $B_c$  transits into a hybrid state  $b\bar{c}g$  where  $b\bar{c}$  stays in a color-octet, by emitting a gluon, and then the hybrid turns into  $B_c(1s)$  by emitting the second gluon. The two gluons eventually hadronize into two pions. In the transitions  $B_c(ns) \to B_c(ms) + \pi + \pi$ , the momentum transfer is not large and the perturbative method does not apply. The QCD multipole expansion (QCDME) method suggested by Gottfried, Yan and Kuang [4–8] properly treats the lightmeson emission process. In the picture of the QCD multipole expansion, the two emitted gluons are not energetic particles, but described by a chromo filed of TM or TE modes [9]. It is worth emphasizing again that the two gluons are not free gluons in the sense of the perturbative quantum field theory, but a field in the QCD multipole expansion. It is easy to understand that such transition is dominated by the E1-E1 mode, while the M1-M1 mode is suppressed for the heavy quarkonia case.

The two chromo-E1 transitions are dealt with in the regular framework of multipole-expansion. The key point is how to properly obtain the mass-spectrum and wavefunction of the intermediate hybrid state. Isgur and Paton[10] suggested to use a modified potential to describe the hybrid states, but there are a few free parameters to be determined. Before, the parameter in the potential[11] which Kuang and Yan employed, was fixed by assuming the  $\psi(4040)$  to be the hybrid [5]. Thanks to the achievements of BELLE, CLEO and BES, an abundant database on such two-pion emission processes has been available. In our previous work [12], we carefully discussed the cases of  $\Upsilon(ns) \to \Upsilon(ms) + \pi + \pi$  and  $\psi(ns) \to \psi(ms) + \pi + \pi$  in terms of the QCD multipole-expansion, then by the method of minimizing  $\chi^2$  which

is widely adopted in analyzing experimental data, we eventually fix the parameters in the potential. For the hybrid state we used three different potential models[10, 13, 14] and noticed that the potential form proposed by Allen  $et \, al.$ [14] better coincides with the lattice result. According to the potential parameters gained by fitting the spectra of heavy quarkonia  $\Upsilon$  and  $\psi$  families, one can estimate the parameters for the members of the  $B_c$  family, by slightly varying the values of corresponding parameters (see below for details).

In this work, we apply the QCD multipole expansion method [5] and the potential form suggested by Allen et al. [14] to calculate the transition rate of  $B_c(ns) \to B_c(ms) + \pi\pi$  where the potential parameters for the hybrids  $|b\bar{c}g\rangle$  ( $|b\bar{c}g\rangle$ ) are the same as that we obtained for  $|b\bar{b}g\rangle$  and  $|c\bar{c}g\rangle$ . Since the spectra and wavefunctions of higher exited states of  $B_c$  are even harder to be accurately derived, at this stage, we only concern the transitions from the radially excited states  $B_c(3s)$  and  $B_c(2s)$  into the lower states via emitting two pions.

The derivation and numerical computations are straightforward and very similar to the procedures carried out in literature, therefore, below unless necessary for clarity, we omit some technical details. This work is organized as follows. After the introduction, we present the theoretical formulae for the transition  $B_c(ns) \to B_c(ms) + \pi \pi$ , and then in sec.III, we list our numerical results along with all input parameters in tables, the last section is devoted to our discussion and conclusion.

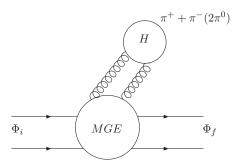


FIG. 1: The two-gluon emission diagram and the two gluons eventually hadronize into two pions. The intermediate state is a hybrid state  $|b\bar{c}g>$ .

#### II. FORMULATION

#### A. The transition width

The theoretical framework about the QCD Multiploe Expansion method is well framed in Refs[5–8], and all the concerned formulas are presented in those papers. Here we only copy a few formulas which are necessary for evaluating the widths of  $B_c(ns) \to B_c(ms) + \pi + \pi$  (n > m) in this work. For all details, the readers are suggested to refer to the original works and references therein. The general formula for the rate caused by double E1 transitions was given in Ref.[5] as

$$\Gamma = \delta_{l_{i}l_{f}}\delta_{J_{i}J_{f}}(G|C_{1}|^{2} - \frac{2}{3}H|C_{2}|^{2}) \mid \sum_{l}(2l+1) \begin{pmatrix} l_{i} & 1 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l & 1 & l_{i} \\ 0 & 0 & 0 \end{pmatrix} f_{i,f}^{l}|^{2}$$

$$+(2l_{i}+1)(2l_{f}+1)(2J_{f}+1) \sum_{k}(2k+1)[1+(-1)^{k}] \begin{cases} s & l_{f} & J_{f} \\ k & J_{i} & l_{i} \end{cases} H|C_{2}|^{2}$$

$$\mid \sum_{l}(2l+1) \begin{pmatrix} l_{f} & 1 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l & 1 & l_{i} \\ 0 & 0 & 0 \end{pmatrix} \begin{cases} l_{i} & l & 1 \\ 1 & k & l_{f} \end{cases} f_{i,f}^{l}|^{2}, \tag{1}$$

where  $l_{i(f)}$  and  $J_{i(f)}$  are orbital and total angular momenta of the initial (final) heavy quarkonia and the total spin of the initial and final states respectively. l is the angular momentum of the color-octet intermediate state.  $|C_1|^2$  and  $|C_2|^2$  are constants to be determined which come from the hadronization of gluons into pions. G and H are the phase-space integrals whose concrete forms were given in [5, 6]. Obviously the first term corresponds to an S-wave and the second term to a mixing of S and D-waves.

In terms of Eq.(1) the transition rate of a pseudoscalar meson into another pseudoscalar meson with a two-pion emission can be written as

$$\Gamma(n_i^{\ 1}S_0 \to n_f^{\ 1}S_0) = |C_1|^2 G|f_{if}^l|^2,\tag{2}$$

which is similar to that between two vector-quarkonia[5, 6].  $n_i, n_f$  are the principal quantum numbers of initial and final states and  $f_{if}^l$  is the overlapping integration over the concerned hadronic wave functions,

$$f_{i,f}^{l} = \sum_{K} \frac{\int R_{f}(r) r^{P_{f}} R_{Kl}^{*}(r) r^{2} dr \int R_{Kl}^{*}(r') r'^{P_{i}} R_{i}(r') r'^{2} dr'}{M_{i} - E_{Kl}},$$
(3)

where  $P_i$ ,  $P_f$  are the indices related to the multipole radiation, for the E1 radiation  $P_i$ ,  $P_f=1$  and l=1.  $R_i$ ,  $R_f$  and  $R_{Kl}$  are the radial wave functions of the initial and final states,  $M_i$  is the mass of initial quarkonium and  $E_{Kl}$  is the energy eigenvalue of the intermediate hybrid state. The sum over the principle number of the intermediate hybrid state K is truncated at K=7 because the contributions from higher excited states are too small and can be safely neglected.

Because strong interaction is blind to flavor and electrical charge, the whole scenario should be applicable for the  $B_c$  cases.

# B. The phenomenological potential

In this study we employ generalized Cornell potential[15] which includes a spin-related term [16] for the initial and final states as

$$V(r) = -\frac{\kappa}{r} + br + V_s(r) + V_0, \tag{4}$$

where  $\kappa = \frac{4\alpha_s(r)}{3}$  and the coupling  $\alpha_s(r)$  can be treated as a phenomenological constant while calculating the spectra of quarkonia. Thus, for the phenomenological application, one does not need to consider their QCD running. The spin-related term  $V_s$  is,

$$V_s = \frac{8\pi\kappa}{3m_O^2} \delta_{\sigma}(r) \overrightarrow{S}_Q \cdot \overrightarrow{S}_{\overline{Q}'}, \tag{5}$$

with

$$\delta_{\sigma}(r) = \left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r^2},\tag{6}$$

and  $V_0$  is the zero-point energy[12].

The intermediate state between the two gluon-emissions is a hybrid state, namely the  $Q\bar{Q}'$  resides in a color-octet. It was indicated that one still can use an effective potential to describe the color-octet  $Q\bar{Q}'$  state [10, 11, 13, 14]. In literature, there are four different effective potential forms which are respectively suggested by the authors of Refs.[10, 11, 13, 14]. In our earlier work [12] we employed the three models of them to study the hybrid state  $|Q\bar{Q}'g\rangle$  (Q stands as b or c in [12]) whereas, Yan and Kuang [5] used the potential given

by Buchmüller and Tye [11]. We find that the potential form suggested by Allen *et al*.[14] coincides with the lattice result better than the others, thus we will use that potential in this work.

The corresponding potential form is

$$V_i(r) = \frac{\kappa'}{8} + \sqrt{(b'r)^2 + 2\pi b'} + V_0'. \tag{7}$$

Because the authors of Refs.[10, 13, 14] did not consider the spin-related term, we have modified the potential by adding the spin-related term  $V_s$  which has the same form given above in Eq.(5), then the potential becomes:

$$V(r) = V_i + V_s. (8)$$

With this modification, we can investigate the spin-splitting effects.

#### III. NUMERICAL RESULTS

At first we need to fix the parameters in the potentials. In our earlier work[12], we re-fitted the spectra of the quarkonia to obtain the corresponding potential parameters in Eq.(4). The values of the parameters are listed in Table I.

TABLE I: The potential parameters for  $c\bar{c}$  and  $b\bar{b}$ 

	$\kappa$	$b(\text{GeV}^2)$	m(GeV)	$\sigma({\rm GeV^2})$	$V_0({ m GeV})$
$c\bar{c}$	0.67	0.16	1.78	1.6	-0.6
$b\bar{b}$	0.53	0.16	5.13	1.7	-0.6

One can see that the parameters b and  $V_0$  in Eq.(4) are the same for  $b\bar{b}$  and  $c\bar{c}$ , so that we suppose they are unchanged for  $b\bar{c}$ . Since the difference between the values of the parameter  $\sigma$  for  $b\bar{b}$  and  $c\bar{c}$  is small, it is plausible to choose  $\sigma=1.65$  for  $b\bar{c}$  ( $b\bar{c}$ ). In the calculation for quarkonia  $m_Q$  is the mass of the constituent quark (b or c), instead, for  $b\bar{c}$  one should use the reduced mass. Here we set  $m_c=1.78$  GeV and  $m_b=5.13$  GeV for concrete numerical

TABLE II: our prediction on mass of some  $b\bar{c}$  mesons

	1s	2s	3s
$0^{-}(\text{GeV})$	6.276	6.880	7.254
$1^- (GeV)$	6.356	6.908	7.274

computations. Fitting the mass of  $B_c(0^-)$  6.276 GeV [17], we obtain  $\kappa = 0.58$ . Then with the given potential we predict the masses of a few other  $b\bar{c}$  states listed in the following table.

For the hybrid potential (7), the strategy in our earlier work [12] is that we use the minimal  $\chi^2$  method which is widely adopted in analysis of experimental data, to determine the potential parameters which are listed in Table III. One can see that b' and  $V'_0$  in Eq.(7) are the same but  $\kappa'$  is different for  $b\bar{b}g$  and  $c\bar{c}g$ . Because no direct measurements on transitions  $B_c^{(*)}(ns) \to B_c^{(*)}(ms) + \pi + \pi$  have ever been conducted so far, we cannot determine the parameter  $\kappa'$  for  $b\bar{c}g$  in terms of available data as we did for the heavy quarkonia. However one can expect that  $\kappa'$  should fall in the region between the values for the  $b\bar{b}g$  and  $c\bar{c}g$  systems, thus we will vary this value slightly within the range to study a possible dependence of the numerical results on  $\kappa'$ . The dependence is shown in Table IV (see below).

TABLE III: potential parameters for  $c\bar{c}g$  and  $b\bar{b}g$ 

	$\kappa'$	$b'(\mathrm{GeV^2})$	m(GeV)	$V_0'({ m GeV})$
$c\bar{c}g$	0.54	0.24	1.78	-0.8
$b\bar{b}g$	0.40	0.24	5.13	-0.8

Then there is still a free parameter  $|C_1|^2$  in the decay rate Eq.(2). It is noted that  $C_1^2$  is a factor related to the hadronization of gluons into two pions, so should be universal for  $\psi$ ,  $\Upsilon$  and  $B_c$  meson decays. Because  $C_1$  is fully determined by the non-perturbative QCD effects, it cannot be derived from an underlying principle so far. Thus in Ref.[12] we fixed  $|C_1|^2 = 182.12 \times 10^{-6}$  in terms of the well measured decay width  $\Gamma(\psi(2S) \to J/\psi + \pi + \pi)$ .

With these potential parameters, we solve the Schrödinger equations to obtain wave func-

tions and masses of the initial, final and intermediate states which appear in the overlapping integration  $f_{if}^l$ . Then we can compute the corresponding widths of the concerned modes. As indicated above, in our calculation we vary  $\kappa'$  in a small region from 0.40 ( $\kappa'(c\bar{c}g)$ ) to 0.54 ( $\kappa'(b\bar{b}g)$ ). The corresponding results can be seen in TableIV. For these numerical results, we observe that  $\Gamma(B_c(3S) \to B_c(2S))\pi\pi$  and  $\Gamma(B_c(2S) \to B_c(1S))\pi\pi$  are not very sensitive to the value of  $\kappa'$ , but  $\Gamma(B_c(3S) \to B_c(1S))\pi\pi$  is.

TABLE IV: prediction at  $\sigma = 1.65$  (widths in units of KeV)

$\kappa'$	0.40	0.42	0.44	0.46	0.48	0.50	0.52	0.54
$M_{b\bar{c}g}(\text{GeV})(l=0, K=1)$	7.629	7.630	7.631	7.632	7.633	7.634	7.635	7.636
$M_{b\bar{c}g}(\text{GeV})(l=1,K=1)$	7.800	7.800	7.801	7.802	7.803	7.804	7.805	7.806
$M_{b\bar{c}g}(\text{GeV})(l=1,K=2)$	8.118	8.119	8.120	8.121	8.121	8.122	8.123	8.123
$M_{b\bar{c}g}(\text{GeV})(l=1,K=3)$	8.417	8.418	8.418	8.419	8.420	8.420	8.421	8.422
$M_{b\bar{c}g}(\text{GeV})(l=1,K=4)$	8.700	8.701	8.701	8.702	8.703	8.703	8.704	8.704
$M_{b\bar{c}g}(\text{GeV})(l=1, K=5)$	8.970	8.971	8.971	8.972	8.972	8.9735	8.973	8.974
$M_{b\bar{c}g}(\text{GeV})(l=1,K=6)$	9.230	9.230	9.230	9.231	9.231	9.232	9.232	9.233
$M_{b\bar{c}g}(\text{GeV})(l=1, K=7)$	9.479	9.480	9.480	9.481	9.481	9.481	9.482	9.483
$\Gamma(B_c(3S) \to B_c(2S))\pi\pi$	11.01	11.07	11.14	10.90	10.95	10.99	11.06	11.11
$\Gamma(B_c(3S) \to B_c(1S))\pi\pi$	4.91	3.95	2.98	5.64	4.85	4.01	3.05	2.38
$\Gamma(B_c(2S) \to B_c(1S))\pi\pi$	64.13	64.01	63.90	63.71	63.60	63.48	63.37	63.25

# IV. OUR CONCLUSION AND DISCUSSION

Study on  $B_c$ ,  $B_c^*$  mesons and their radial and angular excited states is important because they are the last heavy mesons and are composed of two different heavy flavors. Because of their special structures, a thorough study on the production and decay processes where  $B_c$ and its excited states are involved may shed more light on the fundamental interactions, especially the non-perturbative QCD, moreover, may provide some hints to new physics beyond the standard model. Therefore, this field attracts attentions of theorists and experimentalists of high energy physics. The main obstacle for the study is that the production rate of  $B_c$  is small as Chang and his collaborators indicated [1]. However, as LHC begins running, the high luminosity would provide sufficiently large data sample, moreover, a proposed Z-factory with a luminosity much higher than the LEP-I, would offer a clean environment for the  $B_c$  research.

Among all the decay modes,  $B_c(ns) \to B_c(ms) + \pi + \pi$  (n > m) is a favorable one for investigating the  $B_c$  family because it is a strong-interaction process, moreover, it is also an ideal place to study the heavy hybrids  $|b\bar{c}g\rangle (|\bar{b}cg\rangle)$ . Of course, the radiative decay  $B_c^* \to B_c + \gamma$  is also a place to study the family [18], but as our experimental colleagues suggest, at LHCb, the  $\gamma$  detection would be difficult. Instead, the two charged pions are easy to be identified at LHCb detector. A rough estimate of the mass of  $B_c^*$  in terms of the potential model indicates that the mass of  $B_c^*$  is 6.36 GeV which is close to that given in Ref. [19]. It only 80 MeV heavier than the ground state  $B_c$ , thus the mode  $B_c^* \to B_c + \pi$  is forbidden by the final phase space.

For the decay  $B_c(ns) \to B_c(ms) + \pi + \pi$  (n > m), the dominant mechanism is the two-gluon emission which eventually hadronize into two pions. As is indicated in the literature[20], the other minor mechanisms such as the subsequential pion emissions, may interfere with the amplitude induced by the two-gluon emission mechanism to change the lineshape of the differential width. But the total width is definitely determined by the two-gluon emission, so that our numerical results would give the decay width which can be measured in the future experiments at LHCb and Z-factory. The effects induced by the secondary mechanisms, may be measured at the Z-factory where a cleaner environment can provide an opportunity to conduct accurate measurements including the geometrical distribution of produced pions.

In our calculations, we need to input several potential parameters to calculate the masses of the excited states of  $B_c(ns)$  (n > 1). Unlike for the charmonia  $\psi$  and  $\Upsilon$  families, lack of data on their masses causes errors in our numerical results.

We employ the QCD-multipole expansion (QCDME) method to calculate the corresponding widths. The specific potential forms for  $B_c$  meson and hybrid  $b\bar{c}g$  are selected based on the lattice results. With the potential and concerned parameters we find that the mass of the ground hybrid  $b\bar{c}g$  is  $7.63 \sim 7.64$  GeV as the quark content  $b\bar{c}$  is in a spin-singlet. The widths

of  $B_c(3s) \to B_c(2s) + \pi\pi$  and  $B_c(2s) \to B_c(1s) + \pi\pi$  are not sensitive to the change of parameters within the concerned range and it is similar to the cases for  $\Gamma(\Upsilon(ns) \to \Upsilon(ms) + \pi\pi)$ . While calculating the function  $f_{if}^l$ , we sum over the intermediate states of appropriate quantum numbers and truncate the expansion at K = 7. Our calculation suggests that the decay widths of  $B_c(ns) \to B_c(ms) + \pi + \pi$  can reach a few of tens of KeV which is of the same order as  $\psi(ns) \to \psi(ms) + \pi\pi$  and  $\Upsilon(ns) \to \Upsilon(ms) + \pi\pi$ , therefore with the luminosity of LHCb and the proposed Z-factory, there would be no problem to make relatively accurate measurements on such pion radiative decays.

 $B_c$  mesons were marginally produced at the LEP-I, as the luminosity of the proposed  $Z^0$  can be at least three orders higher than that of LEP-I, there should be sufficient data on  $B_c$  available. Moreover, there is a large phase space for the excited states  $B_c(ns)$  (n > 1), so their production rates are not suppressed by the phase space and are similar to that for the ground  $B_c$ , so that there should be a good chance to observe  $B_c(ns) \to B_c(ms) + \pi + \pi$  (n > m) at the  $Z^0$  factory. As aforementioned, the background at the  $Z^0$  factory is relatively small and a clean environment is expected.

It is believed that LHCb and even TEVATRON possess sufficient database for observing such decays. In fact,  $B_c$  production was first observed at TEVATRON, and with the energy and luminosity of LHCb, observation of  $B_c(ns) \to B_c(ms) + \pi + \pi$  (n > m) is definitely feasible. However, on other aspect, both LHC and TEVATRON are hadron colliders, so the background is much messier. With the efforts of the experts including theorists and experimentalists, it is already possible to clearly distinguish the signal from the background. Definitely, high quality generators are necessary for analyzing all possible sources of background [21]. By contrary, the background at the  $Z^0$  factory is not so serious, i.e. the QCD contamination is relatively alleviated, even though it still exists. Detailed analysis on the possible background is a rather difficult job and usually is done by experts. When preparing the manuscript, we have consulted with our experimental colleagues about the possibility of observing such decays and analysis on the background, and here we can only make a very rough discussion.

Therefore, for getting a better understanding of  $B_c$  meson and its excited states, the  $Z^0$  factory is definitely superior to the hadron colliders.

No doubt, the present work is still a primary effort to find the excited states of  $B_c$  and study their structures, as well as that for the hybrid states  $|b\bar{c}g\rangle$ . One can be convinced that the order of magnitude of the numerical results is trustworthy, so that it is optimistic that measurements on such processes at LHCb and even the proposed Z-factory can be conducted. When the data are available, we will be able to further investigate the structure of the  $B_c$  family and identify the mechanism(s) which governs the transitions. Then more precise theoretical works will be needed.

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